

High momentum entanglement of cold atoms generated by a single photon scattering

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With the mechanism of pairwise scattering of photons between two atoms, we propose a novel scheme to highly entangle the motional state between two ultracold neutral atoms by a single photon scattering and detection. Under certain conditions, it is shown that an arbitrary amount of entanglement can be obtained with this scheme.

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Introduction.— Quantum entanglement, as one of the central topics in quantum mechanics, has been extensively studied on the Hilbert spaces with continuous variables (CV) in recent years [1, 2, 3, 4, 5, 6, 7, 8]. Compared to the finite dimensional entanglement on photonic polarizations or atomic internal states, the CV system provides entanglement in a rich diversity of forms [2, 3, 4, 5, 6], which provide unique roles in quantum information processing [1, 7] and in fundamental test for quantum physics [2]. It is only in CV systems that the unbounded high degree of entanglement becomes possible, which has been studied recently for optical squeezed state [1], optical parametric down conversion [3, 4], atom-photon scattering [5], and atom-atom motional state [6], etc..

For motional entanglement between neutral atoms without interactions, it is verified that [6] one can not produce entanglement beyond 1 ebit with single-photon emission and detection, therefore, it unavoidably requires nonlinear bi-photon or multi-modes detections for high entanglement [6]. By introducing effective interactions between atoms in this letter, however, we find that it is possible to produce arbitrarily high entanglement by only a single photon scattering and detection, which is

more feasible and efficient for realistic implementations [7]. The basic physics for this scheme is that, for a couple of cold atoms with the same electric dipoles, photon scattered from one atom can be efficiently re-absorbed and re-scattered by the other, this mechanism of “pair-wise scattering” [9, 10] of photons entangles the atomic wavepackets under the law of momentum conservation. Evaluated by the Schmidt number K [4, 5], it is shown that an arbitrary amount of entanglement can be produced for ultracold atoms with proper physical control parameters.

Theoretical analysis.— To describe the physical process, as shown in Fig. 1, it is assumed that a pair of identical cold atoms “a” and “b” are coupled to a single photon in a cavity along y-axis, with nearly parallel atomic electric dipoles $\vec{d}_{a,b}$, i.e., $|\vec{d}_a| = |\vec{d}_b| = d$ and their relative angle $\varphi \approx 0$. We assume the coupling photon is scattered and therefore recoils the atoms to the x-direction where the photon detector is placed. Considering the momentum exchange along x-direction, the Hamiltonian can be written as $\hat{H}_{\text{total}} = \hat{H}_0 + \hat{H}_I$, where

$$\begin{aligned} \hat{H}_0 &= \frac{(\hat{P}_x^a)^2}{2m} + \frac{(\hat{P}_x^b)^2}{2m} + \sum_{k,s} \hbar \omega_k a_{k,s}^\dagger a_{k,s} \\ &\quad + \hbar \omega_{21} (\sigma_{22}^a + \sigma_{22}^b) + \hbar \omega_c a^\dagger a, \\ \hat{H}_I &= \hbar \sum_{k,s} [g_a(k,s) e^{-ikx_a} \sigma_{12}^a a_{k,s}^\dagger \\ &\quad + g_b(k,s) e^{-ikx_b} \sigma_{12}^b a_{k,s}^\dagger + \text{H.C.}] \\ &\quad + \hbar (g_c \sigma_{12}^a a^\dagger + g_c \sigma_{12}^b a^\dagger + \text{H.C.}). \end{aligned} \quad (1)$$

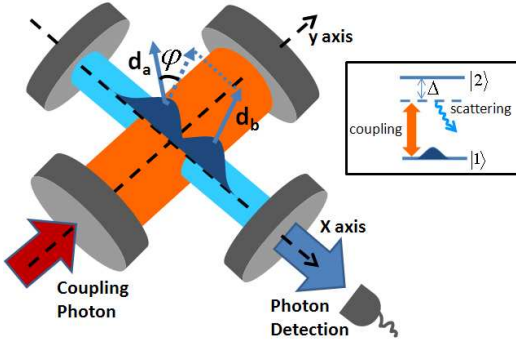


FIG. 1: (Color online) Schematic plot of the physical model. The coupling photon is injected in a single-mode cavity along the y-axis, whereas the scattering photon is collected and detected along x-axis. The recoil of atoms is restricted in the x-direction. The inset shows the atomic configuration.

$\hat{P}_x^{a(b)}$ is the x-dimensional momentum operator for the two-level atom “a” (“b”) with mass m and transition frequency ω_{21} , and $\sigma_{ij} \equiv |i\rangle\langle j|$. a^\dagger (a) is the creation (annihilation) operator for the coupling single mode with frequency ω_c and coupling strength g_c ; whereas $a_{k,s}^\dagger$ ($a_{k,s}$) is for the scattering modes along x-axis with frequency $\omega_k = c|k|$ and polarization s and the coupling strength is $g_{a(b)}(k,s) = \sqrt{\frac{c|k|}{2\varepsilon_0\hbar V}} \cdot \vec{d}_{a(b)} \cdot \hat{\epsilon}_{k,s}$.

We utilize the Dirac ket to denote the physical state, e.g., $|1_a, q_a; 1_b, q_b; k\rangle$ represents that the atom “a” (“b”)

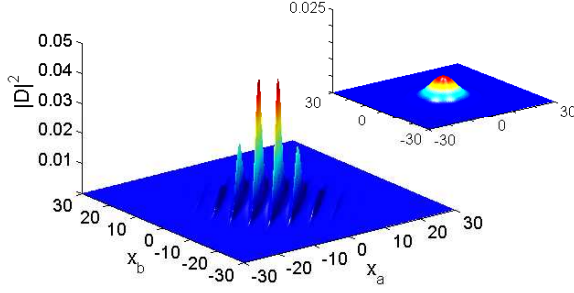


FIG. 2: (Color online) Spatial probability distribution of the steady bipartite wavefunction $D(\mathbf{x}, k=k_c)$, with $k_c=1$, $\sigma=0.2$, $\delta=0.1$ and $E_m=0$ [11]. The small figure shows the initial Gaussian distribution.

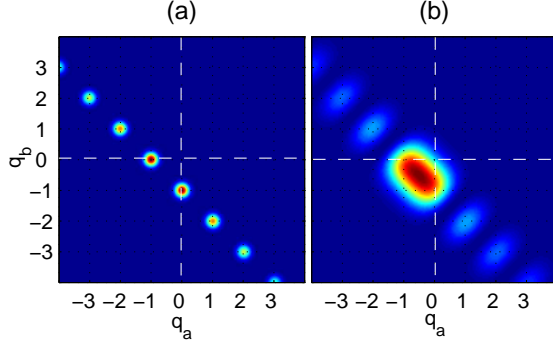


FIG. 3: (Color online) Density plot of momentum distributions for the steady bipartite wavefunctions $D(\mathbf{q}, k=k_c)$ with $k_c=1$, $\delta=0.1$, and $E_m=0$ [11]. (a) Different orders of scattered wavepackets are well separated with $\sigma=0.3$. (b) Wavepackets begin to overlap and interfere with $\sigma=1$.

has the internal state $|1\rangle_{a(b)}$ and momentum $\hbar q_{a(b)}$, and the photon is scattered with momentum $\hbar k$. In the slowly-varying frame, therefore, the system state can be expanded as:

$$\begin{aligned} |\Psi\rangle = & \int d\mathbf{q} e^{-i(E/\hbar + \omega_c)t} \left[A(\mathbf{q})|1_a, q_a; 2_b, q_b; 0\rangle \right. \\ & + B(\mathbf{q})|2_a, q_a; 1_b, q_b; 0\rangle + C(\mathbf{q})|1_a, q_a; 1_b, q_b; 0\rangle \Big] \\ & + \int d\mathbf{q} dk e^{-i(E/\hbar + \omega_k)t} D(\mathbf{q}, k)|1_a, q_a; 1_b, q_b; k\rangle, \quad (2) \end{aligned}$$

where the bold symbol \mathbf{q} simplifies the pairwise variables (q_a, q_b) , and the summation over the polarization s is implied in the integration over k . The kinetic energy is $E \equiv \hbar^2(q_a^2 + q_b^2)/2m$.

In Eq. (2), $D(\mathbf{q}, k)$ is the momentum wavefunction for the atoms and the scattered photon, which is our main concern for its induced atom-atom entanglement in the steady state. $A(\mathbf{q})$ [$B(\mathbf{q})$] is the instantaneous wavefunction for the atom “b” (“a”) is excited, and we introduce $M(\mathbf{q}) \equiv A(\mathbf{q}) + B(\mathbf{q})$ for the symmetry. From

the Schrödinger equation, the dynamical equations read:

$$\begin{aligned} \frac{dM(\mathbf{q})}{dt} = & -i\Delta M(\mathbf{q}) - 2ig_c^* C(\mathbf{q}) - \gamma M(\mathbf{q}) \\ & - e^{iE_mt/\hbar} \int dk \int_0^t ds [g_a(k)g_b(k) \times \\ & e^{i(\omega_c - c|k|)(t-s)} M(q_a+k, q_b-k, s)], \quad (3) \end{aligned}$$

$$\frac{dC(\mathbf{q})}{dt} = -ig_c M(\mathbf{q}), \quad (4)$$

$$\begin{aligned} \frac{dD(\mathbf{q}, k)}{dt} = & -i \exp[-i(\omega_c - c|k|)t] \times \\ & [g_b(k)A(q_a, q_b+k) + g_a(k)B(q_a+k, q_b)], \quad (5) \end{aligned}$$

where the detuning $\Delta = \omega_{21} - \omega_c$ and $\gamma = 2\pi \int dk |g_{a(b)}(k)|^2 \delta(\omega_k - \omega_c)$ is the atomic natural linewidth, and $\Gamma \equiv 2|g_c|^2 \gamma / \Delta^2$ denotes the scattering rate. $E_m = \hbar^2 k_c^2 / m$ represents the kinetic energy mismatch generated when the coupling photon is scattered from one atom to another and be re-scattered back to the coupling mode [9, 10]. When this energy deficit is significant, the pairwise scattering happens only in a time scale $\tau < \hbar/E_m$ and further cascaded processes will be eliminated [9]. In this work, we assume E_m is small enough to allow needed cascaded pairwise scattering, and treat it as a constant [11].

The fourth term at the r.h.s. of Eq. (3) represents the pairwise scattering between two atoms. If the atomic dipoles are perpendicular ($\varphi = \pi/2$), this term vanishes and the pairwise scattering will be forbidden in the Eqs. (3–5), which therefore reduce to the model of a single atom scattering [5]. When the dipoles are parallel ($\varphi \approx 0$), however, photon scattered by one atom can be efficiently re-scattered by the other, therefore produces momentum correlation between them.

Eqs. (3–5) can be analytically decoupled in the atomic position coordinates. We use the argument $\mathbf{x} \equiv (x_a, x_b)$ for wavefunctions to indicate their fourier counterparts, e.g., $M(\mathbf{x}) \equiv \int d\mathbf{q} e^{-i\mathbf{x} \cdot \mathbf{q}} M(\mathbf{q})$, where $\mathbf{x} \cdot \mathbf{q} \equiv x_a q_a + x_b q_b$. With weakly coupling conditions: $g_c, \gamma \ll \Delta$, and $E_m/\hbar \ll \Gamma$, the adiabatic solution of Eq. (5) yields:

$$\begin{aligned} D(\mathbf{q}, k, t) = & N \int d\mathbf{x} e^{i\mathbf{x} \cdot \mathbf{q}} G(\mathbf{x}) (e^{ix_a k} + e^{ix_b k}) \times \\ & [1 - e^{-t \cdot \Pi(\mathbf{x}, k, t)}] / \Pi(\mathbf{x}, k, t), \quad (6) \\ \Pi(\mathbf{x}, k, t) = & i(\omega_c - 2|g_c|^2/\Delta - c|k|) \\ & + \Gamma[1 + \text{sinc}(E_m t/\hbar) \cos(\varphi) \cos(k_c x_a - k_c x_b)], \end{aligned}$$

where $G(\mathbf{x})$ is the fourier counterpart of the initial atomic momentum wavefunction $G(\mathbf{q}) = \sqrt{2} \exp[-(q_a^2 + q_b^2)/\sigma^2] / \sqrt{\pi} \sigma$ which is generally specified as an unentangled Gaussian with momentum width σ . For atoms with nearly parallel dipoles ($\varphi \approx 0$), we introduce $\cos(\varphi) \equiv 1 - \delta^2$ and use $\delta (\ll 1)$ as a dipole parallelity parameter in the following. N is the normalized factor and $\text{sinc}(x) \equiv \sin(x)/x$.

Generation of atom-atom entanglement with single-photon detection—Eq. (6) is the time evolution of collective atoms–photon wavefunction, which will be projected to an atom–atom entangled state [6] after the detection of the scattered photon. In the following, we neglect the light shift ($2|g_c|^2/\Delta$) and assume the photon clicks on a narrow band detector with wavevector $k = +k_c$.

When $E_m \neq 0$, from Eq. (6), the steady atomic momentum wavefunction reduces to a Bell-like state:

$$D(\mathbf{q}, k = k_c, t \rightarrow \infty) \propto G(q_a + k_c, q_b) + G(q_a, q_b + k_c), \quad (7)$$

since for $t \gg \hbar/E_m$ the energy-conserved single-atom-scattering process prevails [9], which can produce at most 1 ebit by a single-photon detection [6].

For $E_m = 0$ [11], however, the atom–atom pairwise scattering introduces effective interactions between them, which produces correlated wavepacket as:

$$D(\mathbf{q}, k = k_c, t \rightarrow \infty) \propto D'(q_a + k_c, q_b) + D'(q_a, q_b + k_c), \quad (8)$$

$$D'(\mathbf{q}) = \exp\left[-\frac{(q_a + q_b)^2}{2\sigma^2}\right] \exp\left(-\frac{|q_a - q_b|\delta}{\sqrt{2}k_c}\right) \times \quad (9)$$

$$\sum_{n=0}^{\infty} \exp\left[-\frac{\sigma^2 \pi^2}{8k_c^2}(2n+1)^2\right] \cos\left[\frac{\pi}{2k_c}(2n+1)(q_a - q_b)\right].$$

Eq. (9) characterizes the essences for the pairwise scattering: the first Gaussian factor on the r.h.s. represents the bipartite total momentum which is provided by the initial momentum width σ ; the Fourier series on the second line, which gives a periodic structure of the relative momentum, characterizes different orders of pairwise scattering, with the total number of scattering orders controlled by the dipole parallelity parameter δ through the second Gaussian factor on the first line.

The spatial and momentum distributions of the atomic wavefunction of Eq. (8) are plotted in Fig. 2 and Fig. 3, respectively. In Fig. 2, one sees that, the scattering of photon with wavevector $k = k_c$ correlates the atomic wavepackets with resonant spatial period λ_c , and produces grating-like wavefunction with highly localized atomic relative positions. Meanwhile, as in Fig. 3, the atomic momentum is correlated along $q_a + q_b + k_c = 0$ due to the momentum conservation. When the initial atomic momentum width σ is well below the recoil momentum ($\sigma \ll k_c$) as in Fig. 3 (a), different orders of scattered wavepackets are clearly separated, which corresponds to a well entangled bipartite state; when σ tends towards k_c , however, as in Fig. 3 (b), recoiled wavepackets begin to overlap and exhibit destructive quantum interferences for the high-order scattering, which will eventually destroy the momentum entanglement that can be seen more clearly in the following.

The entanglement encoded in the wavefunction Eq. (8) can be quantitatively evaluated by the Schmidt decom-

position [8] of $D(\mathbf{q}, k=k_c)$:

$$D(\mathbf{q}, k=k_c) = \sum_{n=-\infty}^{\infty} \sqrt{\lambda_n} S_n(\mathbf{q}), \quad (10)$$

where $S_n(\mathbf{q})$ is the momentum Schmidt basis which is orthonormal and separable, with $\sum_n \lambda_n = 1$. The degree of entanglement is then defined as the Schmidt number [4, 5]: $K \equiv 1/\sum_n \lambda_n^2$.

For the single-atom-scattering state in Eq. (7), compared with Eq. (10), one has at most $K = 2$ which is only 1 ebit [6]. For the pairwise-scattering-state in Eq. (8), however, from the numerical results of K in Fig. 4 in dependence of δ and σ , one sees that it may highly exceed the 1 ebit limit [6] for cold atoms ($\sigma < k_c$) with parallel dipoles ($\delta \approx 0$).

For atoms cooled well below the recoil temperature, i.e., $\sigma \ll k_c$, the Fourier series in Eq. (9) makes the relative momentum highly localized in each single period, therefore, the summation over spatial frequency can be well approximated by a summation over discrete localized momentum modes. Along with Eqs. (8–10), it eventually yields the Schmidt decomposition as a summation of different orders of scattered wavepackets:

$$S_n(\mathbf{q}) = (-1)^{|n|+1} \text{Sgn}(n) G(q_a - nk_c, q_b + nk_c + k_c), \quad (11)$$

$$\lambda_n = N' \left(e^{-\sqrt{2}\delta|n|} - e^{-\sqrt{2}\delta|n+1|} \right)^2, \quad (12)$$

where $\text{Sgn}(n)$ is the signum function with value $+1$ for $n \geq 0$ and -1 for $n < 0$, and N' is a normalization factor. With Eq. (12), the degree of entanglement is obtained:

$$K \equiv 1/\sum_n \lambda_n^2 = \sqrt{2}/\delta, \quad (\delta \ll 1). \quad (13)$$

Eq. (13) fits well with the numerical results when $\sigma < k_c/2$ as shown in Fig. 4. It indicates that, by utilizing ultracold atom pair, such as in BEC, where both conditions $\delta \approx 0$ and $\sigma \ll k_c$ are well fulfilled, arbitrarily high atom–atom entanglement may be produced by a single photon scattering and detection, once the mismatch energy E_m is well compensated [11].

For hotter atoms with temperature approaches or exceeds the recoil temperature, or equivalently, $\sigma \gtrsim k_c$, the scattered momentum wavepackets $S_n(\mathbf{q})$ will overlap each other and exhibit quantum interferences. From Eq. (11), one sees that different orders of recoiled wavepackets take the same shape as the initial wavepacket $G(\mathbf{q})$, together with an extra overall phase: $(-1)^{|n|+1} \text{Sgn}(n)$. Explicitly, the first two orders of scattered wavepackets $S_0(\mathbf{q})$ and $S_{-1}(\mathbf{q})$ take the same overall phase “ -1 ”, and therefore exhibit constructive interference; for higher-order wavepackets, however, $S_n(\mathbf{q})$ take opposite phases “ ± 1 ” interchangeably, which will induce destructively interferences as in Fig. 3 (b). Due to this quantum interference, the entanglement will be significantly decreased

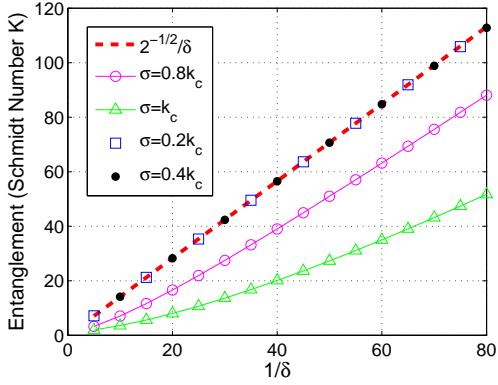


FIG. 4: (Color online) The Schmidt number K for the entangled steady state of $D(\mathbf{q}, k = k_c)$ is plotted against the reciprocal dipole parallelity parameter $1/\delta$, with different initial momentum widths σ as shown in the inset.

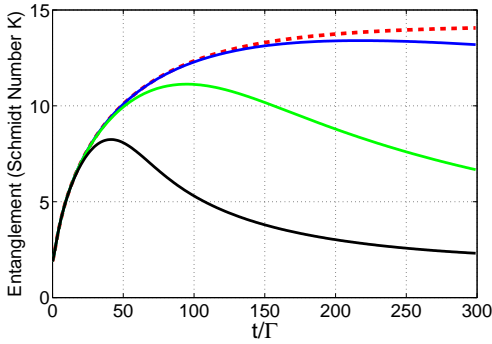


FIG. 5: (Color online) Time evolution of the produced entanglement. The dashed red line indicates $E_m/\hbar = 0$ [11], whereas the solid lines from top to bottom are plotted with $E_m/\hbar = 0.1 \times 10^{-3}$ (blue), $E_m/\hbar = 0.5 \times 10^{-3}$ (green), $E_m/\hbar = 2 \times 10^{-3}$ (black), respectively. $\Gamma = 1$, $\sigma = 0.2k_c$, $\delta = 0.1$.

as in Fig. 4, therefore, one can hardly produce high entanglement for hot atoms with $\sigma \gg k_c$, even when the conditions $\delta \approx 0$ and $E_m = 0$ [11] are fulfilled.

The time evolution of the produced atomic entanglement can be obtained from the general solution in Eq. (6). As in Fig. 5, the steady entangled state is established with a time τ which can be estimated as $\tau \sim 1/(\delta^2\Gamma)$. For $\hbar/E_m \gg \tau$ [11], the maximal entanglement can be obtained by a single photon detection with the coupling time T satisfying $\tau \ll T \ll \hbar/E_m$. For the general case of $E_m \neq 0$, the produced entanglement will first increase due to the pairwise scattering, and then decrease to a Bell-like state ($K \approx 2$) when $T \gg \hbar/E_m$, since the energy-conserved single-atom scattering process dominates in this time scale [9].

Conclusion.— From the first principle we have demonstrated that, the mechanism of pairwise photon-

scattering in a couple of ultracold atoms may be used to produce superhigh atom-atom entanglement. When the atom pair is cooled well below the recoil temperature with parallel electric dipoles, it is possible to achieve an arbitrary amount of entanglement by a single photon detection once the scattering-induced energy mismatch E_m is compensated [11]. This scheme can be used to produce entangled atom resources for the test of EPR-nonlocality [2] and for quantum information processing [1, 7].

For experimental tests of this scheme, the spatially overlapping ultracold atom pairs can be produced by coupling weak atom laser beams from a BEC, and then be loaded into a cavity for detections of the scattered photon [12]. Moreover, as in recent report [10] of BEC superradiant [9], the generation of pairwise scattering by atom pairs is preferred over single atom scattering when the energy mismatch E_m is compensated by the incident coupling light [10], therefore, it is most probable to analyze the quantum correlation in the pairwise scattering process with this proposed model.

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